Why Evasion Is Not Recommended? a Game Theory Model.

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***Abstract* — The decision about tax declaration is a decision under uncertainty. The failure to report one´s full income to the government does not induce automatically to a penalty, it takes time, investigation, and extra costs.**
In this project, we are considering a simple model where this decision is the only one with which the taxpayer is concerned. So, we showed the strategies that the government can apply to avoid tax evasion.

I. INTRODUCTION

From the tax reforms implemented in Mexico, is has been observed that, unfortunately, taxpayers are more likely to evade their tax responsibilities[i]. The tax payment results from a combination of duty and imposition: "I pay because it is my obligation"; "I pay because if I don't, the authorities will impose me a sanction".

The soul of this article does not pretend to show numerical results; instead, it carries out conclusions as a consequence of a theoretical mathematical procedure.

The preferences/interests of the authorities and the taxpayers are often opposed. The government is interested in collecting as much tax as possible while the citizen prefers to pay as little as possible and obtain the greatest benefits.

The problem of tax evasion is usual and unfortunately attractive. This problem has occasioned many economic and social lags.

This work, using Game Theory, shows an interaction between individuals and the government, each with their respective strategies: the government auditing and individuals evading or not. The Game Theory is an analytical tool that captures interactions among the various agents of a society through their behavior.

This strategic behavior has been applied to real situations: competition between firms analyzed by Tirole[ii], or auctions as present Vickrey[iii], or to problems of cooperation as show Fudenberg and Masking[iv].

In this work, we use a Game Theory model to present the simultaneous decisions of the taxpayer and the government. Neither of them knows the strategies of the other. With this, we seek to find the optimal strategies of each one, and ultimately, we focus on the optimal strategies the government could, at some point, use. Then, we faced to a problem: the government does not have always a dominant strategy. So we work on the scenarios where the government may apply an audit without falling in a situation where the costs of this one exceed the punishment that will be imposed to the taxpayer.

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II. METHODOLOGY

Due to this problem, it is possible to represent the behavior of the taxpayer and the government with a model of Game Theory. First we began with the next definition,

Definition1. A game is a strategy interaction between economics agents, called players. This game is represented in the following way[v],

$$\Gamma = \{N, A_i X A_j, U(a_i, a_j)\}$$

Where the number of players is N, A_i is the set of strategies of player i, a_i is the strategy of the player i and $U(a_i, a_j)$ is the payment function.

The representation of a game in normal form is shown in "Diagram 1", where the rows represent the strategies of player 1, and the columns represent the strategies of player 2. The matrix's elements are the payment functions.

	c	d
a	$u_1(a,c), u_2(a,c)$	$u_1(a,d), u_2(a,d)$
b	$u_1(b,c), u_2(a,c)$	$u_1(b,d), u_2(a,c)$

Diagram1. The representation of a game in normal form.

Definition2. a_i^* is dominant strategy for i, if[vi]

$$U_i(a_i^*, a_j) > U_i(a_i', a_j)$$
$$\square\ a_i' \varepsilon A_i \ \ and \ \ a_j \varepsilon A_j$$

Speaking of the payment function, we are interested in utility functions, which represent welfare.

For example, if such function depends on consumption (C) and indirectly on wealth (W) or income; this is, $U(C)$ and $C = f(W)$, where the wealth W can depend of other parameters or variables endogenous in the model.

Definition3. A utility function must satisfy the following conditions[vi],

i) $\frac{dU(C)}{dC} > 0,$

ii) $\frac{d^2U(C)}{dC^2} < 0.$

The first condition means that as the income increases, the well-being is bigger. The second condition shows that welfare increases at slower rates than consumption does. Graphically, a utility function is presented in "Figure 1".

One of the most used utility functions in economic theory and that satisfies both conditions previously mentioned is $U(x) = \ln(x)$ with $x > 0$.

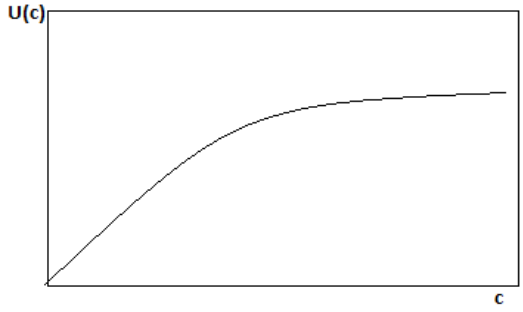

Figure 1. A utility function satisfying the conditions i) and ii).

III. DEVELOPMENT: THE GAME

We have a situation where two players are involved. First we have a taxpayer that *must* pay taxes based on its full income. In the other side we have the government who will receive those taxes. The taxpayer can declare taxes based on its full income or based on an amount less than its full income. One of the problems to which the taxpayer faces, is deciding which amount to declare, because if they fail to report one´s full income, the government will apply a penalty, but this won't happen immediately. However, there are incentives to do it.

The government knows that the taxpayer may not declare its full income, so they apply in some cases an audit. Every time that the government applies this audit, the taxes authorities apply a penalty to the taxpayer. There are cases where this penalty takes place due to the evasion of taxes, but there are also cases where this penalty does not take place because of non-tax evasion. It should be added that the audit implies a cost to the government.

For this reason, *the model involve the next variables,*

- W: real income
- X: declared income
- $W - X$: non-declared income,
- θ: tax rate
- π: penalty rate
- k: audit cost

The *Strategies* of taxpayer are,

i) (E) Evade payment of taxes,
ii) (NE) Not evade payment of taxes.

For its part, the government has the next *Strategies*,

i) (T) Trust the taxpayer and not audit it or,
ii) (DT) Distrust the taxpayer and audit it.

Now, we find the payment function. For example, when the taxpayer decides (E) and the government decides (DT), the incomes for this situation are:

Taxpayer: $W - \theta X - \pi(W - X)$

Government: $\theta X + \pi(W - X) - k$

And thus, their payments with this income are,

$U_T(E, DT) = U_T\big(W - \theta X - \pi(W - X)\big)$ **(1)**

$U_G(E, DT) = U_G(\theta X + \pi(W - X) - k)$ **(2)**

Analogously, when the taxpayer decides (E) and the government decides (T), we will have the next payment:

$U_T(E, T) = U_T(W - \theta X)$ **(3)**

$U_G(E, T) = U_G(\theta X)$ **(4)**

Similarly, when the taxpayer decides (NE) and the government decides (DT), we will have the next payment:

$U_T(NE, DT) = U_T\big(W - \theta X - \pi(W - X)\big)$, with X = W **(5)**

$U_G(NE, DT) = U_G(\theta X + \pi(W - X) - k)$, with X = W **(6)**

Finally, when the taxpayer decides (NE) and the government decides (T),

$U_T(NE, T) = U_T(W - \theta X)$, with X = W **(7)**

$U_G(NE, T) = U_G(\theta X)$, with X = W **(8)**

It is important to remark that, the realization of an audit implies a cost (k) to the government. Also, when the taxpayer does not evade taxes, the declared income becomes its real income; then, the penalty does not take place. The game in normal form is seen in the "Diagram 2".

	DT	**T**
E	$U_T(E, DT), U_G(E, DT)$	$U_T(E, T), U_G(E, T)$
NE	$U_T(NE, DT), U_G(NE, DT)$	$U_T(NE, T), U_G(NE, T)$

Diagram2. Game in normal form: strategy interaction between the players.

IV. RESULTS

According to the intuition of the model, the taxpayer always has incentives to evade taxes; the next result shows the conditions when this is satisfied.

Lemma1. Regardless of what the government does, the company always has incentives to evade if the *penalty rate is less than the tax rate*.

Proof1. We must show that,

$U_T(E, DT) > U_T(NE, DT)$ **(9)**

and

$U_T(E, T) > U_T(NE, T)$ **(10)**

This is,

$U_T\big(W - \theta X - \pi(W - X)\big) > U_T(W - \theta W)$ **(11)**

$U_T(W - \theta X) > U_T(W - \theta W)$ **(12)**

Now, by the first condition over $U(X)$ in the "Definition 3", the arguments maintain their relations; this is, of (11)

$$W - \theta X - \pi(W - X) > W - \theta W$$
$$\theta(W - X) - \pi(W - X) > 0$$
$$(W - X)(\theta - \pi) > 0$$
$$(\theta - \pi) > 0$$
$$\theta > \pi$$

And, of course at the assumption in the Lemma, this is true.

Now, of (12)

$$W - \theta X > W - \theta W$$
$$X < W$$

According to the values of the variables, this is true.

Some comments about of the results of lemma,

a) When the government distrust, the taxpayer always evade because is most attractive the monetary amount that fails to pay, than the punishment that will be imposed.
b) The lemma suppose tax rate is greater than the penalty rate, but we can observe that if $\theta \leq \pi$, the taxpayer has no reasons to evade taxes.
c) In game theory, the taxpayer's strategy for evade taxes is a dominant strategy.

Even though this inequality is true when the taxpayer evades taxes, we cannot say that always the taxpayer has incentives to evade taxes, but we can name the cases in which this statement is true. Then, if the government wants to know how much the penalty rate should be worth, it must be ensured to be equal to or greater than the tax rate.

The government, in its Law of Income Tax (96 article)[vii], shows different ranges of monthly tax rate depending on the taxpayer's income. This information is shown in the next table:

Monthly Fee

Lower Limit	Upper Limit	Fixed Fee	Percent Applicable
$	$	$	%
0.01	496.07	0.00	1.92
496.08	4,210.41	9.52	6.40
4,210.42	7,399.42	247.24	10.88
7,399.43	8,601.50	594.21	16.00
8,601.51	10,298.35	786.54	17.92
10,298.36	20,770.29	1,090.61	21.36
20,770.30	32,736.83	3,327.42	23.52
32,736.84	62,500.00	6,141.95	30.00
62,500.01	83,333.33	15,070.90	32.00
83,333.34	250,000.00	21,737.57	34.00
250,000.01	And more	78,404.23	35.00

So θ_i is the "percent applicable in the excess of the lower limit" of each range.

Lemma2. The taxpayer has no incentives for evading if the penalty rate is greater than the difference between different tax rate ranges.

$$\pi > \theta_i - \theta_j, \qquad i > j$$

Proof2. We know that,

$$W = X + (W - X) \tag{13}$$

Then,

$$W\theta_i = [X + (W - X)]\theta_i$$
$$W\theta_i = X\theta_i + (W - X)\theta_i \tag{14}$$

On the other hand,

$$(W - X)\theta_i = (W - X)\theta_j + (W - X)(\theta_i - \theta_j), \quad i > j \tag{15}$$

Thus, we have three cases,

From (14) y (15), with $i > j$

Case I. $\pi = \theta_i - \theta_j$,

$$W\theta_i = X\theta_i + [(W - X)\theta_j + (W - X)\pi] \tag{16}$$

Case II. $\pi > \theta_i - \theta_j$,

$$W\theta_i < X\theta_i + [(W - X)\theta_j + (W - X)\pi] \tag{17}$$

Case III. $\pi < \theta_i - \theta_j$,

$$W\theta_i > X\theta_i + [(W - X)\theta_j + (W - X)\pi] \tag{18}$$

Some comments about the results of this lemma,

a) The Case II is an optimal strategy for the government.
b) If the penalty tax is equal to $\theta_i - \theta_j$, the tax payer has not incentives to evade taxes.

So the government has information about its optimal strategy for making decisions. But questions stand out: is always feasible to apply the optimal strategy of the government? When does the government must apply it and when does not?

Lemma3. The optimal strategy of the government is feasible when the audit cost is smaller than the applied punishment.

$$k < \pi(W - X), \quad k > 0$$

Proof3. If auditing (DT) was a dominant strategy for the government,

$$U_G(E, \mathrm{DT}) > U_G(E, T) \tag{19}$$
$$U_G(NE, DT) > U_G(NE, T) \tag{20}$$

Then, from (19)

$$U_G(\theta X + \pi(W - X) - k) > U_G(\theta X)$$
$$\theta X + \pi(W - X) - k > \theta X$$
$$\pi(W - X) > k$$

But, from (20)

$$U_G(\theta W - k) > U_G(\theta W)$$
$$\theta W - k > \theta W$$
$$\theta W - \theta W > k$$
$$0 > k$$

We fall into a contradiction because the audit cost cannot be smaller than cero.

Some comments the results of this lemma,

a) The government distrusting the taxpayer is not a dominant strategy.
b) When the taxpayer does not evade taxes, the government should not apply an audit.

V. CONCLUSIONS

We have examined a simple model of simultaneous strategies between two players. Based on the assumptions of this text, if the government implements a tax rate greater than the penalty rate, the taxpayer will not have incentives to evade taxes. Even more, if the penalty rate is greater than the difference between different ranges of the tax rate, the taxpayer will not either have incentives to evade taxes.

Knowing that applying an audit is not always a dominant strategy, the government has valuable and backed up information to decide when to apply it.

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